MASS TRANSFER FROM SINGLE SPHERES IN STOKES FLOW WITH SURFACE REACTIONS

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NOMENCLATURE

a,	radius of sphere;
c,	concentration of reacting species;
c_∞,	concentration of reacting species at infinity;
D,	diffusion coefficient;
k,	reaction rate constant;
R,	radial co-ordinate;
r,	dimensionless co-ordinate, $r = R/a$;
U_∞,	fluid velocity at infinity;
α,	dimension parameter defined by $2\alpha = -(2 + \sigma) + [(2 + \sigma)^2 - 4]^{1/2}$;
B_n,	dimensionless parameter defined by $$B_n = \frac{[2(\alpha + 1) + \sigma(n + 1)]}{[\sigma n - 2(\alpha + 1)]};$$
β,	dimensionless parameter defined by $\beta = 1/(1 + \bar{\sigma})$;
θ,	angular co-ordinate;
σ,	dimensionless parameter $\sigma = D/kac_\infty$;
$\bar{\sigma}$,	dimensionless parameter $\bar{\sigma} = D/ka$;
ϕ,	dimensionless concentration $\phi = [(c/c_\infty) - 1]$;
Pe,	Peclet number $= 2aU_\infty/D$;
Sh,	Sherwood number.

THE purpose of this communication is to present improved expressions for the average rate of mass transfer from a sphere in a Stokes type flow with either first or second order surface reactions for small Peclet numbers.

The results were derived by employing the now well known singular perturbation technique of Kaplun [1], Lagerstrom and Cole [2] and Proudman and Pearson [3] to solve the convection equation. Due to the fact that the solution closely parallels the work of Acrivos and Taylor [4] and Taylor [5] in their analysis of heat transfer from a sphere in Stokes flow and slip flow, respectively, details of the derivation are omitted.

The results presented here extend the results of Hartunian and Liu [6], but are in disagreement with those of Frisch [7] since Frisch neglected the effect of free steaming motion of the mass transfer. The title problem has also been considered by Acrivos and Chambré [8, 9] for large Peclet numbers, but the discussion herein will be limited to small Peclet number situations.

The transport equation describing the mass transfer from a sphere in Stokes flow when written in dimensionless variables takes the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) = \frac{Pe}{2}\left[\left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right)\cos\theta\frac{\partial\phi}{\partial r} - \left(1 - \frac{3}{4r} - \frac{1}{4r^3}\right)\frac{\sin\theta}{r}\frac{\partial\phi}{\partial\theta}\right]. \qquad (1)$$

The boundary conditions to be satisfied for a second order chemical reaction are $\phi \to 0$ as $r \to \infty$ and

$$\sigma\frac{\partial\phi}{\partial r} = (\phi + 1)^2 \text{ at } r = 1.$$

For a first order reaction the conditions would be $\phi \to 0$ as $r \to \infty$ and $\bar{\sigma}\dfrac{\partial\phi}{\partial r} = \phi + 1$ at $r = 1$.

The solution for ϕ can be determined in a straight forward manner by the singular perturbation technique mentioned earlier. The results can then be used to calculate the average Sherwood number defined by the formula

$$Sh = -\int_{-1}^{+1}\left(\frac{\partial\phi}{\partial r}\right)_{r=1} \mathrm{d}(\cos\theta).$$

If this analysis is carried out the resulting expressions for the average Sherwood number are found to be

(a) for first order reactions;

$$Sh = -2\left[\beta + \beta^2\frac{Pe}{4} + \frac{\beta^2}{8}Pe^2\ln\left(\frac{Pe}{2}\right) + g\frac{Pe^2}{4} + \frac{\beta^3}{32}Pe^3\ln\frac{Pe}{2} + \ldots\right] \qquad (2)$$

where

$$g = \beta\left[\frac{\beta^2}{4} + \frac{49\beta}{240} - \frac{1}{16(1 + 2\bar{\sigma})} + \frac{3(1 + \bar{\sigma})}{32(1 + 2\bar{\sigma})} + \frac{(1 + 3\bar{\sigma})}{64(1 + 2\bar{\sigma})} - 0{\cdot}08639\right].$$

(b) for the second order reactions;

$$Sh = 2\left[\alpha - \frac{\alpha Pe}{4B_o} + \frac{\alpha(\alpha + 1)}{(\sigma + 2\alpha + 2)}\frac{Pe^2}{4}\ln\left(\frac{Pe}{2}\right) + g(\alpha, \sigma)\frac{Pe^2}{4} + \ldots\right] \qquad (3)$$

where

$$g(\alpha, \sigma) = \frac{1}{(\sigma + 2\alpha + 2)} \left[(2\alpha^2 + 2\alpha) \left(- 0{\cdot}25723 - \frac{1}{4B_o} \right) - \frac{3\sigma\alpha}{8} - \frac{\sigma \bar{K}}{3B_1} - \frac{\alpha^2}{4} \left(1 + \frac{1}{B_o} \right)^2 - \frac{1}{3} \left(\frac{\bar{K}}{B_1} - \frac{3\alpha}{8} \right)^2 - \frac{10\bar{K}(\alpha + 1)}{24B_1} \right] + \frac{3\alpha}{8} + \frac{\bar{K}}{3B_1}$$

$$\bar{K} = \frac{3}{8} \frac{[2\alpha^2 + \alpha(2 + 3\sigma)]}{[\sigma - 2(\alpha + 1)]}. \tag{3b}$$

From these two results we see that the change in reaction order effects only the coefficients of the series expansion in Peclet number. Therefore it appears that for all integer reaction orders the expansion for the Sherwood number in terms of the Peclet number retain the same form except for different coefficients. Also it is important to note that both of these results appear to remain valid for the entire range $0 \leqslant \sigma \leqslant \infty$ and $0 \leqslant \bar{\sigma} \leqslant \infty$. In conclusion we observe the interesting fact that the Sherwood number for a second order reaction is a function of the free stream concentration, but such is not the case for a first order surface reaction.

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